



Mathemagic

- *Highlighting* -



Educational Goals

- ❖ Use the place value to solve a mathematical situation
- ❖ Use the decomposition of numbers to solve a mathematical situation

Key Features of the Targeted Competencies

- ❖ To decode the elements of the situational problem
- ❖ To apply different strategies in order to elaborate the solution
- ❖ To validate the solution
- ❖ To justify actions or statements by referring to mathematical concepts and processes
- ❖ To define the elements of the mathematical situation
- ❖ To mobilize and apply concepts and processes appropriate to the given situation

Concepts Used

- ❖ Place value (tens, hundreds, units)
- ❖ Arithmetic (addition, subtraction)

Materials

- ❖ Video of the trick
- ❖ 1 full deck of cards (54 cards) or 54 objects to manipulate

Targeted Academic Level
Grades 4-6

Mathematical Field Concerned



Suggested Teaching Formula



Time Required
Approximately 35 minutes



Suggested Process



The goal of the exercise is to let the students discover the magician's trick

Step 1: Introduction (5 minutes)

Play the video of the magic trick once (www.amazingmaths.ulaval.ca).

Note: Using cards is not necessary; we simply need 54 objects easily distributable.

Step 2: Recreate the magic trick (10 minutes)

It's your turn to do the trick in front of your students! You will find in the Explanation Sheet for the trick "**Highlighting**" the steps to follow to do the trick.

It is a good idea, before beginning, to make the students understand that each one of the manipulations is essential to the **trick's success** and that they must be very **attentive to the magician's instructions**.

Since this trick is done in group, some adjustments to the **Explanation Sheet** must be done:

1. Stand where all the students can see you, without being close to a board. Select 4 assistants that will choose numbers in their head. These students remain in their seat, so the rest of the group can follow the trick.
2. Before you turn around, choose a student that will distribute the cards to the 4 assistants. This step is important for you to not know the number of cards distributed. Remind the students to not reveal their number aloud.
3. The student that distributed the cards brings you back the remaining cards.
4. Mention that you will count these cards. Meanwhile, ask the 4 assistants to come to the board to do the addition of the 4 numbers, with the class' supervision. Remind the students to not say the answer aloud and to clearly identify it.
5. Reveal your answer and then turn around.

Step 3: Finding the solution (15 minutes)

Achieve the solution in group. Guide the student's thoughts to make them understand the 4 following aspects:

Aspect 1:

No matter what the numbers chosen by the assistants are, the number of the first assistant will always have 1 in the tens position, the number of the second assistant will always have 2 in the tens position, the number of the third assistant will always have 3 in the tens position and the number of the fourth assistant will always have 4 in the tens position.



Suggested Process



Lines of thinking:

- What was each assistant supposed to do when he took his cards? *(Take the number of cards corresponding to the number of tens and take the number of cards corresponding to the units.)*
- What can you notice about the choice of numbers of each spectator; do they have a characteristic in common? What do the numbers between 10 and 19 have in common, the ones between 20 and 29, etc.? *(They have the same number of tens: assistant 1 always takes 1 card to represent his number of tens, assistant 2 always takes 2 cards to represent his number of tens and so on.)*

Aspect 2:

We always have 10 cards representing the total of tens. So, the sum of the tens is always 100 units. *(1 ten = 10 units. 10 tens that are each worth 10 units = 100 units.)*

Lines of thinking:

*Ask the assistants to take back the cards that were taken, **but only for the tens.***

- What is the number of cards taken by the assistants for the number of tens in this case? *(1 + 2 + 3 + 4 = 10)*
- Is this information always true, no matter what the chosen numbers are? *(Yes)*
- 10 cards are distributed for the tens. How many units does this represent? *(100 units)*

Aspect 3:

The remaining cards can indicate us the total of units.

Lines of thinking:

- How many cards did we have at the beginning of the trick? *(54)*
- How many cards do we distribute for the tens? *(10)*
- How many cards are left for the units' distribution? *(54 - 10 = 44)*

There are 54 cards at the beginning and the assistants respectively take 1, 2, 3 and 4 cards to represent the tens of each number. There are 44 cards left to distribute for the units.

Ask the assistants to take the cards representing the units.

- Why does the magician count the remaining cards? What information can it tell him? *(He can deduce the number of units taken.)*
- Which arithmetic operation does the magician have to do to get the total of units? Why? *(44 - number of remaining cards = total of units taken by the assistants)*

Note: The magician cannot know the units for each of the 4 numbers, but only the total of units.



Suggested Process



Aspect 4:

We have 10 tens totalizing 100 units and a known total number of units giving us the final sum.

- What does the magician have to do to finally know the final sum? ($100 + \text{the total of units}$)
- Does the magician have to calculate the total of tens every time? (*No, the number of tens is constant.*)

In summary:

The magician knows the total number of cards distributed for the tens (10 cards) and the total number of cards distributed for the units (see the operation above).

Thus, since we know that 10 tens equal 100 units, the magician simply has to add 100 to the answer of the subtraction he did to get the sum of the 4 mystery numbers chosen by the spectators.

To go further!

Do the trick again, but with a different number of objects at the beginning (for example 60 instead of 54). What would be the magician's new trick?

What is the minimum quantity of objects to have for the trick to work for all the choice possibilities of the 4 spectators?