AMAZINGMATHS

MATHEMAGIC

- THE THIEF -

Materials:

- 1 writing slate
- 1 erasable marker
- Several cards with digits 1 to 9

How to do the Magic Trick

Goal:

Find the digit ‘’stolen’’ by the spectator.

Trick:

1. The magician asks a spectator to write a 4-digit number of their choice on a chalkboard (ex: 5 246). The magician turns around so that he cannot see the number.
2. Still without looking, he asks the spectator to choose a different number containing the same digits and to write it underneath the other one.

   Example: 5 246.

   6 452.
3. The spectator subtracts the smallest number from the largest number and writes the answer obtained (ex.: 6 452 – 5 246 = 1 206). The magician asks if the answers contain 0s. If this is the case, the spectator must remove them by erasing them or crossing them out.
4. For every digit of the answer, the spectator selects a card that represents the same number. For example, if the answer is 1 206, the spectator takes cards 1, 2 and 6. The zero has been eliminated; we do not take it into account. He then places the cards on the table.
5. The magician asks the spectator to ‘’steal’’ one of the cards. He must keep it in his hands without showing it. The magician will find the ‘’stolen’’ digit!
6. The magician turns around and observes the digits in front of him. *(In his head, he calculates the sum of the remaining digits, and he subtracts this result by 9. If the sum is a two-digit number, these digits must be added again to get an answer with only one digit. This is the digit we subtract from 9).*
7. The magician announces what digit the spectator has ‘’stolen’’.

MATHEMATICAL EXPLANATION

Why this trick works

This trick is based on the divisibility rule by 9: a number is divisible by 9 when the sum of its digits is itself divisible by 9.

> Note: Another way to use the rule is to make the <u>repetitive sum</u> of the digits that make up the number: if the sum of the digits of the initial number is composed of several digits, the sum of the digits is repeated, and so on, until a <u>single digit</u> is obtained. If this last digit is 9, then the number is divisible by 9. Otherwise, the number is not divisible by 9.
>
> Ex.: Is 9 654 divisible by 9?
>
> We have: 9 + 6 + 5 + 4 = 24.
>
> Since 24 has 2 digits, we repeat the operation: 2 + 4 = 6.
>
> 6 is a single digit. Since it is not 9, then 9 654 is not divisible by 9.
>
> Note: Refer to ''Why Does the Divisibility Rule by 9 Work?'' to understand this reasoning.

Let's explain how the trick works by proceeding with an example:

The spectator chooses the numbers: 2 476 and 7 264

He subtracts the smallest number from the greatest number.

7 264 (greatest number) – 2 476 (smallest number) = 4 788.

Then, the spectator takes a card for each of the digits of the number, which represents the same number. If the answer contains a 0, the spectator must remove it from the answer.

$\boxed{4}$ $\boxed{7}$ $\boxed{8}$ $\boxed{8}$

Then, he ''steals'' a card that he keeps in his hands. This is the digit that the magician will find.

$\boxed{4}$ $\boxed{7}$ $\boxed{8}$

The magician is able to predict the stolen digit thanks to the following mathematical result:

> The magician knows that the difference between two numbers that contain the same digits is divisible by 9, so that the <u>repetitive sum</u> of the digits that compose it is equal to 9.
>
> Note: Refer to ''Why is the different between two numbers that contain the same digits a multiple of 9?''.

He is missing a digit to get to 9. In order to find the stolen digit, the magician makes the repetitive sum of the digits he has in front of him. By subtracting 9 from his answer, he determines the missing digit so that the sum is 9 again (this is the stolen digit).

Thus, we have: 4 + 7 + 8 = 19 (19 having 2 digits, we repeat the operation) (1 + 9 = 10) (1 + 0 = $\boxed{1}$

Since 9 – $\boxed{1}$ = 8, we know that 8 is missing for the number to be divisible by 9. So, the stolen digit is 8.

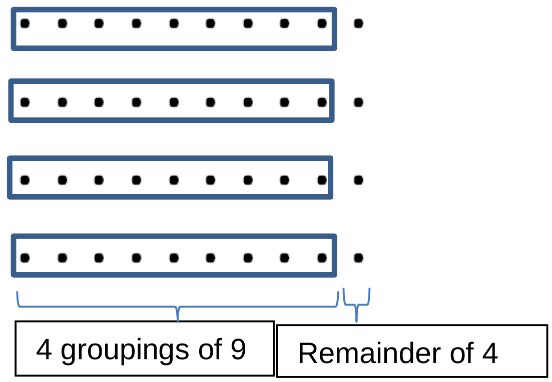
Note: if the repetitive addition of the digits is equal to 9, the spectator then stole a 9 or a 0. Since 0s are removed, the spectator has a 9 in his hands.

MATHEMATICAL EXPLANATION

Why Does the Divisibility Rule by 9 Work?

To understand the notion of divisibility by 9, we must use the notion of division with a remainder. To make it simpler, it is good to visualize the division by 9 as groupings of 9 units with a certain remainder lower than 9. For example:

40 = 4 x 9 + 4 (4 groupings of 9 with a remainder of 4)



Now, let's group the powers of 10 into groups of 9.

10^0 = 1 = 0 x 9 + 1 (0 groups of 9 remainder 1)

10^1 = 10 = 1 x 9 + 1 (1 group of 9 remainder 1)

10^2 = 100 = 11 x 9 + 1 (11 groups of 9 remainder 1)

10^3 = 1000 = 111 x 9 + 1 (111 groups of 9 remainder 1)

Etc.

Note that the remainder of the powers of 10, when divided by 9, is always 1.

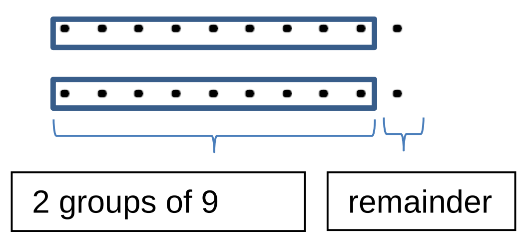
Also, when adding the digits composing the same number, we add the number of units, tens, hundreds, etc. that our number possesses.

Thus, every unit, every ten, every hundred, every thousand can actually be represented as a certain number of 9-unit groups with a remainder of 1. From this, we conclude that by adding the digits that make up a number, we add the number of remainders 1 that the number possesses when divided by 9. For example,

326 = 300 + 20 + 6

300 = 3 x 100
As mentioned earlier, 100 = 11 x 9 + 1 (11 groups of 9 remainder 1).
So, 3 x (11 x 9+1) = 33 x 9 + 3 (33 groups of 9 remainder 3).

20 = 2 x 10
As mentioned 10 = 1 x 9 + 1 (1 group of 9 remainder 1)
So, 2 x (1 x 9 + 1) = 2 x 9 + 2 (2 groups of 9 remainder 2).



6 = 6 x 1
Since 1 = 0 x 9 + 1 (0 groups of 9 remainder 1)
So 6 X (0 x 9 + 1 = 0 x 9 + 6 (0 groups of 9 remainder 6)

MATHEMATICAL EXPLANATION

Why Does the Divisibility Rule by 9 Work? (continued)

Thus,

326 = 300 + 20 + 6
 = 33 groups of 9 <u>remainder 3</u> + 2 groups of 9 <u>remainder 2</u> + 0 groups of 9 <u>remainder 6</u>
 = 35 groups of 9 <u>remainder 11</u>

However, the remainder is greater than 9. So, we can still make groups of 9 with the same technique.

11 = 1 group of <u>9 remainder 1</u> + 0 groups of 9 <u>remainder 2</u>
 = 1 group of <u>9 remainder 2.</u>

Thus, we find that the remainder of 326, when divided by 9, is 2.

326 = 35 groups of 9 + 1 group of <u>9 remainder 2.</u>
 = 36 groups of 9 <u>remainder 2.</u>

$$\begin{array}{r|l} 326 & 9 \\ \hline -27 & 36 \\ \hline 56 & \\ -54 & \\ \hline 2 & \end{array}$$

Remainder 2

In other words, we always find that in a number the <u>repetitive sum</u> of the digits that compose it gives the remainder of this number's division by 9. If the remainder is 0, it means that this number is divisible by 9!

This is also why if the sum of the digits composing a number is a multiple of 9 the number will be divisible by 9!

MATHEMATICAL EXPLANATION

Why is the difference between two numbers that contain the same digits a multiple of 9?

We know that the sum of the digits composing these numbers is the same since they have the same digits and the addition is commutative. Thus, we deduce that the remainder of the division by 9 of the two numbers is the same. Let's take an example:

1. 7 264
2. 2 476

First, let's calculate the remainder of these numbers when we divide them by 9 (let's do the repetitive sum of the digits that make them up).

$$7+2+6+4 = 2+4+7+6 = 19 \Rightarrow \quad 1+9=10 \Rightarrow \quad 1+0=1$$

Thus, we deduce that the remainder of the two numbers when divided by 9 is 1.

1. 7 264 = 807 x 9 + 1 (807 groups of 9 remainder 1)
2. 2 479 = 275 x 9 + 1 (275 groups of 9 remainder 1)

When we subtract these two numbers, the remainders being the same, they will be eliminated, so we will only have groups of 9.

7 264	→	807 groups of 9 remainder 1
− 2 476	→	− 275 groups of 9 remainder 1
4 788	→	532 groups of 9 no remainder

Since we only have groups of 9 forming the difference, then it is necessarily divisible by 9.